Method of synthesis of flexible strategies for preventing collisions

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ABSTRACT: Flexible strategy is the method, which allows forming a strategy of the operating vessel for avoiding collision, when navigating in congested waters and the risk of collision with more than one vessel exists. This strategy does not conflict with the ColReg requirements and take into account multi-variant approach of conduct of targets and possibility of worsening of situation in time. The methodological base for the solution of this problem is the theory of the dynamic *n*-guided systems.

For analytical description of the process of manoeuvering in congested waters for preventing collision, the methods of theory of the dynamic *n*-guided systems were used. This theory is the logical evaluation of differential game theory, which is applied by many authors in the synthesis of models for the search of optimum manoeuvers for collision avoidance.

Now we will show that the process of manoeuvering of several ships in confined waters for collision avoidance is adequately described by the methods of the theory of dynamic *n*-guided systems. The dynamic system defines any object or process for which the initial state is determined as aggregate of any variables and constants values and a function law, which describes the change of the initial state in time and can be used for the prognosis of the future state of the dynamic system. In the case when the state of the dynamic system depends on input control influences of a few participants, i.e. input control influence is distributed between the participants, and to every participant the non-empty set of strategies of control is put in accordance, such dynamic system is called as *n-guided.* Depending on the aims of participants' control, cooperation between them can carry the co-operative, antagonistic, coalition or other type of interaction.

For the process of manoeuvering of a few vessels in confined waters, assume that in some area of control S_c there are n vessels. This area of control S_c is fixed with a definite ship, which we will name as an *operating vessel*. The local proximity of ships belonging to the control area allows to consider their aggregate, as some dynamic system which must be described from positions of their safe relative motion. Such dynamic system Σ is controlled by all n vessels.

The state of system Σ is described by the coordinates $\xi_i(t)$ and $\eta_i(t)$ of each ship and their parameters of movement $V_i(t)$ and $K_i(t)$ in the two dimensional rectangular system of coordinates, related to the area of control S_c and oriented in N-S direction. Therefore the state $x=x(t)$, as element of set of X, is a 4n – measured vector. The change of the state x of the system Σ is described by differential equations of the ship movement, taking into account their inertia descriptions.

$$\dot{x}(t) = \begin{vmatrix} \dot{x}_1(t) \\ \ldots\ldots \\ \dot{x}_i(t) \\ \ldots\ldots \\ \dot{x}_n(t) \end{vmatrix} = \begin{vmatrix} \ldots\ldots \\ \dot{\xi}_i \\ \dot{\eta}_i \\ V_i \\ K_i \\ \ldots\ldots \end{vmatrix} = \begin{vmatrix} \ldots\ldots \\ V_i \sin K_i \\ V_i \cos K_i \\ f_v(V_{3i}, \beta_i) \\ f_k(\beta_i) \\ \ldots\ldots \end{vmatrix}$$

The dynamic system Σ is characterized by two additional factors. At first, each of the ships of the system Σ is proceeding by planned route, i.e. expedient activities of each of the ships of the system. And, secondly, strategies for collision avoidance D_i of ships depend on each other and on current position. If dependence exists, it is necessary to get its formal description. The second factor determines the type of interaction, arising between ships at dangerous approaching, expressed in the applied strategies for collision avoidance.

This factor (type of interaction between ships) is the most essential for providing safe passing, and from the point of formalization is the most indefinite. In all cases, a basic normative document regulating the conduct of ships when the risk of collision exists is the ColReg, which foresee coordination of only binary interactions of the pair of ships. When operating in congested waters, manoeuvering can be limited by existing dangers for navigation and more than two vessels can be involved in the risk of collision. Even in the situation of the meeting of two vessels in the open sea the ColReg generate the row of ambiguousness. So, each of the meeting vessels shall not only define the presence of dangerous situation, but also define the range where she "may take action" (Rule 17(a)(ii)), and where she "shall take action" (Rule 17(b)). Each of the vessels involved as estimation can get different ranges of mutual duties at the same beginning position. Therefore, observing the ColReg requirements, the vessels are forced to make decision of involving risk of collision and choice of proper strategy for safe passing in the conditions of considerable vagueness.

The necessity of formalization of the interaction of ships in the conditions of existing risk of collision has defined the features for analytical description of the dynamic *n-guided* system Σ, containing vessels.

For the purpose of analytical formalization of the presence of collision situation in the system Σ we input the concept of *situational disturbance*. Situational disturbance in relation to an operating vessel arises when a ship cannot continue realization of programmed trajectory of motion due to existing risk of collision with one or a few ships of the system Σ. The possibility of the situational disturbance appearance is determined as a result of prognosis of the state of the system Σ for any time period. If the forecast trajectory in the space of states of the system Σ is safe, situational disturbance is absent. Otherwise if there is situational disturbance, the necessity of its compensation appears.

The space of positions $M(P_n)$, the description of which are distances between the ships of the system Σ, consists of a few subsets $M(P_{nk})$, each of which is characterized by some levels of risk of collision. For every pair of ships four subsets of their relative position are determined: the subset of safe positions P_{n0} and three subsets of positions P_{n1}, P_{n2}, P_{n3} with a different degree of danger situation of collision (in accordance with the number of ranges of mutual duties according to Rule 17 of ColReg). Thus in each of the subsets the proper type of conduct is prescribed to the pair of interactive ships. Indicated representation of the space of positions $M(P_n)$ allows to formalize the concept of situational disturbance. In general, situational disturbance between the pair of dangerous ships is offered to be characterized by initial ω_{ijn} and maximal ω_{ijmx} intensity, which can take on a whole number values depending on the range of mutual duties in the initial moment and at the moment of time of the CPA (closest point of approach). Consequently, initial intensity ω_{ijn} can take on values from 0 to 2, and maximum – from 1 to 3. When the dynamic system Σ consists of more than a pair of ships, the situational disturbance is described by the square matrix D_{bn} of n dimension, the element of which d_{ij} is initial ω_{ijn} and maximum ω_{ijmx} intensity.

$$D_{bn} = \begin{Vmatrix} 0 & \omega_{12n},\omega_{12mx} & \dots & \omega_{1nn},\omega_{1nmx} \\ \omega_{21n},\omega_{21mx} & 0 & \dots & \omega_{2nn},\omega_{2nmx} \\ \dots & \dots & 0 & \dots \\ \omega_{n1n},\omega_{n1mx} & \omega_{n2n},\omega_{n2mx} & \dots & 0 \end{Vmatrix}$$

In the case when situational disturbance is produced from an operating vessel, that disturbance is the vector, got from the line of matrix, which corresponds to the operating vessel.

$$D_{bn} = \begin{vmatrix} \omega_{12n},\omega_{12mx} & \omega_{13n},\omega_{13mx} & \dots & \omega_{1nn},\omega_{1nmx} \end{vmatrix}$$

Thus components of vector of situational disturbance, relating to the ships, with which operating vessel is passing clearly at a safe distance is equal to zero. Otherwise they contain the values of ω_{ijn} and ω_{ijmx}.

The nature of the situational disturbance is in the forecast of finding the targets in PAD (predicted area of danger). As a matter of fact, estimations of the pair of situational disturbances (ω_{ijn}, ω_{ijmx}) and (ω_{jin}, ω_{jimx}) are not always symmetric for both ships. It is provided by subjective individual authentication of initial range of mutual duties of each ship, as a result of which different areas S_{nd1} and S_{nd2} can be got. Therefore the presence of the situational disturbance may be possible for one of the ships, and at the same time may not exist for another.

Situational disturbance exposes the advent of dangerous position in advance, according to the prognosis of the change of relative position of ships. Therefore it has conditional character, because possible actions of ships and the method of prognosis the change in the dynamic system state influences on the truth of its realization.

The appearance of situational disturbance generates interaction between ships, and there is the task of the disturbance compensation by the choice of the proper strategy for preventing collision. The structure of interaction between the ships of the dynamic system Σ is uniquely determined by the structure of the situational disturbance. The type of interaction between ships determines different system states of the dynamic system Σ. As a result of the researches made three system states of the system Σ were found, each of which is characterized by separate system property.

So, if the number n_b of the interactive ships is equal to zero, i.e. n_b=0, there is no situational disturbance in the dynamic system, interaction of ships is absent, and the elements of the dynamic system (ships) are unrelated. They execute presence of a special purpose functions, realizing the programmatic trajectories (planned route) of motion. The dynamic system Σ is in the initial (first) system state, which is characterized by independent differential equalizations, describing the controlled motion of ships along the planned route.

In this case the dynamic system Σ is characterized by the zeroing matrix of situational disturbance, which contains zeroing elements only.

$$D_{bn1} = \begin{Vmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & 0 & \dots \\ 0 & 0 & \dots & 0 \end{Vmatrix}$$

For the system Σ the absence of interaction between ships is principally important. The structure of the system Σ is characterized by the absence of interaction between elements, which determines its first system property.

For the second system state of the dynamic system Σ the matrix of situational disturbances D_{bn} contained the «isolated» elements $d_{ij,}$ which does not equal zero, thus no more than one on the line of matrix D_{bn}.

$$D_{bn2} = \begin{Vmatrix} 0 & \omega_{12n}, \omega_{12mx} & \dots & 0 \\ \omega_{21n}, \omega_{21mx} & 0 & \dots & 0 \\ \dots & \dots & 0 & \dots \\ 0 & 0 & \dots & 0 \end{Vmatrix}$$

It means that there is only independent conduct of pair of ships in the dynamic system Σ.

In this case situational disturbance converted the dynamic system Σ into a new system state, because a new (second) system property appears. A new system property characterized by relative position and interaction appears in the pair of ships, the parameters of distance appear between ships and bearing from a ship to the ship, which did not exist for a separate ship. Note should be taken on that appearance of the indicated system property which is conditioned by the change of the system Σ structure - origin of interaction between elements. Thus the structure of relations has the following feature: the separate ship of the dynamic system Σ can be conducted only with one ship of the system.

In this case there is only binary interaction, in which strategy of compensation gets out depending on intensity of situational disturbance, and the parameters of the manoeuvre for collision avoidance are counted. Thus differential equalizations describing the motion of the pair of interactive ships are linked between themselves and, producing the common management of position, ships, joined by some rule, compensate situational disturbance, changing the strategy of expedient motion to the strategy of situational disturbance compensation.

The dynamic system Σ appears in the third system state, if one of ships in the system will interact with more than one partner. As it was specified before, the system state of the dynamic system Σ was determined from the analysis of the formed matrix of situational disturbance D_{bn}. A matrix D_{bn} in the beginning is checked up consequently on lines, here the non-zeroing elements of matrix are fixed in every line, if such are present. If even one line of matrix contains two or more of such elements, Σ is in the third system state.

$$D_{bn3} = \begin{Vmatrix} 0 & \omega_{12n}, \omega_{12mx} & \dots & \omega_{1nn}, \omega_{1nmx} \\ \omega_{21n}, \omega_{21mx} & 0 & \dots & 0 \\ \dots & \dots & 0 & \dots \\ \omega_{n1n}, \omega_{n1mx} & 0 & \dots & 0 \end{Vmatrix}$$

If each line of matrix contains no more than one non-zeroing element, it is required to check up all columns of matrix for the presence of non-zeroing elements d_{ij}. In the case when even one column contains more than one nonzeroing element of matrix, the dynamic system Σ is in the third system state.

System Σ gains new system characteristic in the aspect of controllability and indignation, as well as strategy for preventing collision, becomes' complicated, new objects called - *coordinating frameworks* appear which are the consequently-parallel structures of surrounding ships necessary to put in order by complex strategy of situational disturbance compensation. Relations uniting a few elements with each other appear in the structure of the dynamic system Σ.

The matrix of situational disturbance D_{bn} becomes the source of forming situation frameworks linking the structure of dangerous and obstacle ships for a definite vessel. In this case the groups of dependent equalizations of interactive ships are selected from the independent initial system of differential equalizations, for which it is required to find the concerted strategies (interdependent) providing compensation of situational disturbance.

It should be noted that in the third system state the dynamic system Σ_{ns} is characterized by situational disturbances having a matrix form. Therefore for compensation of situational disturbances in this system state the method of external control, which would allow to prang all interactions of situational disturbance simultaneously and in complex, would be the most effective, converting the dynamic system Σ_{ns} into the first system state. However possible realization of management compensating matrix situational disturbance is by the algorithm of joint co-operation of ships, which is not compatible with the principle of coordination, fixed on the ColReg basis. Consequently, while manoeuvering for collision avoidance submitted to operating by the ColReg, realization of effective external management by the dynamic system Σ_{ns} is impossible.

Therefore, compensation of situational disturbance is produced by the forces of interactive ships, thus a definite ship of the system selects the line of the matrix of disturbance, to which she belongs and forms vectorial situational disturbance being the component of the matrix. Thus vectorial situational disturbance includes a few targets interactiving with an operating vessel, and for compensation of this situational disturbance an operating vessel is required to form a strategy of avoiding collision, which must not conflict with the ColReg requirements.

Thus, in general situational disturbance converts the dynamic system Σ from the first unrevolted system state into more revolted states, and the strategy of situational disturbance compensation foresees convertion of the dynamic system Σ into the initial unrevolted state by elimination in the structure of the system of interaction relations between elements (ships).

In the indefinite terms a partner is conduct, when even grounding the probabilistic distributing of choice by the partner of strategy for preventing collision is difficult, a ship is required to use the principle of *flexible strategies* application for preventing collision (compensation of situational disturbances). The sense of this principle consists of the minimax approaches of the ship to the use of possible alternative strategies for safe passing.

We will consider the principle of flexible strategies application for collision avoidance in details. For this purpose we will appeal to the second system state of the dynamic system Σ. Compensation of situational disturbance in this system state of the system Σ is simply regulated by coordinator ColReg, which coordinates strategies for preventing collision of interactive ships. So, if the pair of interactive ships is in the first range of mutual duties, the concerted strategies of avoiding collision $D_1(t_y,K_y)$ and $D_2(Tr_2)$ are prescribed to the ships. The privileged ship c_2 shall keep her course and speed, realizing the programmed trajectory of motion Tr_2 (strategy $D_2(Tr_2)$), and compensation of situational disturbance is produced by a ship c_1 by strategy, which provides safe passing at distance of the shortest approaching, equal to limited-possible distance L_d. For this purpose it is necessary to produce the calculation of the values of time of the beginning of deviation t_y and course of deviation K_y.

$$K_{y(s)1} = K_{oty(s)} + \arcsin[\rho^{-1}\sin(K_2 - K_{oty(s)})]$$
$$K_{y(s)2} = K_{oty(s)} + \pi - \arcsin[\rho^{-1}\sin(K_2 - K_{oty(s)})]$$
$$K_{y(p)1} = K_{oty(p)} + \arcsin[\rho^{-1}\sin(K_2 - K_{oty(p)})]$$
$$K_{y(p)2} = K_{oty(p)} + \pi - \arcsin[\rho^{-1}\sin(K_2 - K_{oty(p)})]$$

where

$$\rho = \frac{V_1}{V_2}$$

$$K_{oty(s)} = \alpha + \arcsin(\frac{L_d}{L})$$

$$K_{oty(p)} = \alpha - \arcsin(\frac{L_d}{L})$$

Here V_1 and V_2 the speed of vessels c_1 and c_2, respectively; L the distance and α is the bearing from the ship to target; K_2 is the course of vessel c_2.

$$t_{yn} = \frac{L_d + L\sin(\alpha - K_{oty(s,p)})}{V_{otn}\sin(K_{otn} - K_{oty(s,p)})}$$

Here K_{otn} and V_{otn} is the initial relative course and speed of target. In general four alterations of courses are possible – two in the opposite direction courses and two in the same direction courses.

If ships are in the second range of mutual duties, unlike the previous situation, the privileged ship c_2 can undertake the manoeuvre of deviation, applying strategy of avoiding collision $D_2(K_y=K_{extr})$, which maximizes minimum distance of the closest approach.

$$t_{yn} = t_n$$
$$K_y = K_1 \pm \arccos(\rho), \quad (\rho < 1)$$
$$K_y = \alpha + \frac{\pi}{2} + \arcsin[\rho^{-1}\cos(K_1 - \alpha)], \quad (\rho \geq 1)$$
$$\max D_{\min} = L|\sin(K_{otextr} - \alpha)|$$

Here K_{otextr} relative course applying strategy of avoiding collision:

$$K_{otextr} = \pi + K_1 \pm \arcsin\rho$$

where K_1 is the course of vessel c_1.

And, finally, if position of ships belongs to the third range of mutual duties, where it is required to take urgent measures for preventing collision, the ColReg orders to both ships to take such action as will best aid to avoid collision. This requirement formalized by the application of strategy

$$D_{1,2}[K_y = \alpha(t) + \pi, t_{y1}, K_{y1} = K_{extr}]$$

regardless of conduct of the second ship, followed on the first stage of manoeuvre by a course equal to the reciprocal bearing on the second ship, and on the second – course of deviation K_{extr} to the output in range of safe positions.

$$t_y = t_n \ ,$$

$$t_{y1} = \frac{L_{d2} - L}{V_1 - V_2 \cos(K_2 - \alpha)} \, .$$

Flexibility of the considered strategies to avoid collision in this system state Σ consists of the following. At first, strategy of collision avoidance as type of ship's conduct for preventing collision changes depending on the realized range of mutual duties. Secondly, compensation of situational disturbance foresees transfer of current position of ships from subset of dangerous positions to subset of safe positions, passing by intermediate subset. So, if compensation of situational disturbance began in the third range of mutual duties, the ship shall take urgent action as will best to avoid collision. Current position of ship must pass the second, and then and in the first range of mutual duties. During this transition strategy of collision avoidance is transformed into the next sequence

$$D_{1,2}[K_y=\alpha(t)+\pi] \rightarrow D_{1,2}(K_y=K_{extr}) \rightarrow D_{1,2}\,(t_y,K_y).$$

And, thirdly, during realization of the required type of strategy for preventing collision, taking into account the high level of vagueness of target conduct, there must be the prepared reserve strategy of avoiding collision, which must be realized at the unforecasted change of current situation.

During compensation of situational disturbance in the third system state of the dynamic system Σ, an operating vessel is in close water situation with a few targets and must choose strategy for preventing collision in general case having a few deviations of course, each of which being carried out by separate component strategy not conflicting with the requirements of coordinator (ColReg).

Therefore in the case when situational disturbance cannot be correctly compensated within the existent requirements of the ColReg, by operating binary co-operations of ships, principle of organization of structure of complex interaction, which can have a few consequences in time levels not conflicting with the ColReg requirements, is offered. Realization of this principle conduces to the concept of *coordinating framework* and development of the method of its forming. In other words, coordinating framework is the instrument of transformation of complex interaction of ships in the well-organized structure of consequences co operations of operating vessel with the group of targets in accordance with the ColReg requirements.

However coordinating framework must be transformed into the real, saving the attained accordance of complex strategy of situational disturbance compensation to the requirements of Rules. Thus, forming of the real framework takes into account manoeuvring of other obstacle ships, dangers for navigation and inertia descriptions of operating ships. The procedure forming the real coordinating framework is the method of synthesis of flexible strategy for preventing collision, which has in component strategies of three above mentioned types for each range of mutual duties of operating vessel and targets of separate level of coordinating framework.

Flexible strategy, as method of the operating vessel conduct for compensation of complex situational disturbance, at the choice of numeral values of parameters is the manoeuvre to avoid collision of ship with targets, thus in offered approach optimization principle of choice of manoeuvre parameters is realized, regardless of the type of component strategies.

Conception of flexible strategies for preventing collision of ships is examined as temporary measure of transitional character, which in further development must result in principle new optimum cooperation control system when risk of collision exists out of hard limits of the restricted binary co-ordination of ColReg. A methodological base for the solution of the outlined problem remains the theory of the dynamic *n–guided* systems with the use of principles of external management and the choice of game co-operative principles of situational disturbances compensation.

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