Analytical model of position uncertainty of ship's plan geometry in integrated navigation system

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ABSTRACT: For safety reason it would be essential to apply uncertainty of ship's contour position in safety evaluation of maneuvers carried out on basis of INS (Integrated Navigation Systems) indications, instead of real dimensions of the ship's contour. The paper presents analytical method of ship's plan geometry (contour) uncertainty area determination. The model was used to determine uncertainty area of ship maneuvering in Świnoujście harbor for typical configuration of navigational equipment applied in existing pilot systems. The results of experiment were discussed. The model equations were derived from measurement error propagation theory. Potential application of uncertainty area as safety zone around ship contour was appointed.

1 INTRODUCTION

Ship's contour, as a geometric object, presented in Integrated Navigation Systems -INS (ECDIS, pilot systems) and determined on basis of measured navigational parameters is affected by some uncertainty. Depending on type of integrated navigation system, the number of factors influencing ship's location changes. The basic parameters the ship's contour position is determined by are her geographical position and true course. The fact that those parameters have a random character the ship's position cannot be identified in the deterministic process and can be expressed by ship's position uncertainty area, which is the area horizontally occupied by ship and its dimensions and can be determined by probabilistic method at assigned confidence level (Tomczak 2006). Distance of uncertainty area outline to navigational obstruction can be considered as criterion of maneuvering safety assessment carried out on the basis of INS indications. The main goal the analytical model should attain is possibility of quick ship's position uncertainty determination for input standard uncertainties of subsystems indications used in integrated system (GNSS position system, heading source). Additionally other input uncertainties are inserted into the model. It results from equipment configuration of the system, version of INS (portable or stationary) and also the place of GNSS antenna location on the ship's deck.

2 MATHEMATICAL MODEL OF SHIP'S LOCATION UNCERTAINTY AREA

The uncertainty area of ship's position is defined by points' coordinates. It is the sum of consecutive points coordinates of real model of the ship's contour and corresponding uncertainties (eq. 1):

$$x_{ni} = x_{ri} \pm c\sigma x_{ri} \quad (1)$$

$$y_{ni} = y_{ri} \pm c\sigma y_{ri}$$

where: x_{ni}, y_{ni} – the consecutive coordinates of points of ship's position uncertainty area for WGS-84 UTM XY projection, x_{ri}, y_{ri} – calculated coordinates of consecutive points of ship's contour, $\sigma x_{ri}, \sigma y_{ri}$ – uncertainties of ship's contour points coordinates measured along x and y of Cartesian axes.

Specifying location of the ship presented in INS can be treated as combined measurement consisting of parameters measured directly (coordinates, heading) and parameters connected with practical solutions the system works in (contour shape approximation, the assessment of GNSS antenna location in the ship coordinate system). In order to examine the accuracy of the position of the ship's outline a mathematical model for the determination of the area of uncertainty of ship's position at any level of probability/confidence has been designed where model of measurement procedure and uncertainty propagation rule have been involved.

One of the most important elements of combined measurement uncertainty assessment procedure is to define the formula for measurement result. Visual model of measurement is presented on figure 1 where $\vec{P_A P_{ri}}$ is a vector between GNSS antenna and consecutive point of ship's contour.

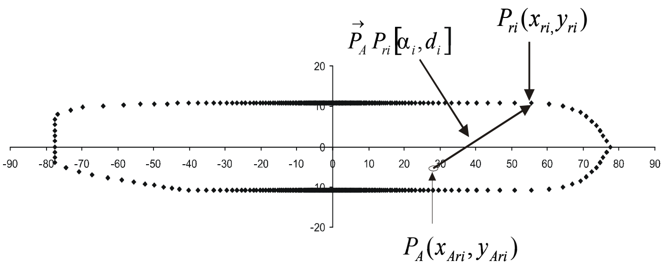

Fig. 1. The visual measurement model of consecutive points of ship's contour

Ship's location in NIS can be determined based on following quotations:

$$x_{ri} = x_{Ari} + d_i \sin(\psi_{ri} + \alpha_i)$$
$$y_{ri} = y_{Ari} + d_i \cos(\psi_{ri} + \alpha_i) \qquad (2)$$

where: x_{ri}, y_{ri} – calculated coordinates of consecutive points of ship's contour, x_{Ari}, y_{Ari} – recorded positions of GNSS antenna – assuming north up orientation, ψ_{ri} – heading, d_i – distance between GNSS antenna and point of ship's contour, α_i – angle between GNSS antenna and point of ship's contour.

Consecutive points coordinates of ship's contour outline (x_{ri},y_{ri}) are measured values, which are two-dimensional random variables described by two-dimensional function vector of many partial random variables. The estimators of measured values (x_{ri},y_{ri}), are calculated from equitation 3 for input estimators x_{Ari}, y_{Ari}, d_i, ψ_i, λ_i for N input values (Sanecki 2004).

$$M_{cov} = \begin{bmatrix} \frac{\partial x_{ri}}{x_{Ari}} & \frac{\partial x_{ri}}{y_{Ari}} & \frac{\partial x_{ri}}{d_i} & \frac{\partial x_{ri}}{\psi_i} \\ \frac{\partial y_{ri}}{y_{Ari}} & \frac{\partial y_{ri}}{x_{Ari}} & \frac{\partial y_{ri}}{d_i} & \frac{\partial y_{ri}}{\psi_i} \end{bmatrix} \cdot$$

$$\begin{bmatrix} \sigma x_{Ari}^2 & \sigma x_{Ari} y_{Ari} & 0 & 0 \\ \sigma x_{Ari} y_{Ari} & \sigma \delta y_{Ari}^2 & 0 & 0 \\ 0 & 0 & \sigma d_i^2 & 0 \\ 0 & 0 & 0 & \delta \psi_i \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial x_{ri}}{x_{Ari}} & \frac{\partial y_{ri}}{x_{Ari}} \\ \frac{\partial x_{ri}}{y_{Ari}} & \frac{\partial y_{ri}}{y_{Ari}} \\ \frac{\partial x_{ri}}{d_i} & \frac{\partial y_{ri}}{d_i} \\ \frac{\partial x_{ri}}{\psi_i} & \frac{\partial y_{ri}}{\psi_i} \end{bmatrix} \qquad (3)$$

Based on general formula of uncertainty propagation theory (eq. 3) the standard uncertainties of input values were determined.

Covariance matrix of two-dimensional probability density function M_{cov} presents equitation 4:

$$M_{cov} = \begin{bmatrix} \sigma x_{ri}^2 & \sigma x_{ri} y_{ri} \\ \sigma x_{ri} y_{ri} & \sigma y_{ri}^2 \end{bmatrix} \qquad (4)$$

where: $\sigma x_{ri} y_{ri}$ -covariance of random variables (x_{ri},y_{ri}).

Multiplying matrixes of equitation 3, combined standard uncertainties of relevant consecutive points coordinates, forming ship's contour shape and its covariance were obtained(eq. 5):

$$\sigma x_{ri} = \sqrt{\sigma x_{Ari}^2 + \sigma d_i^2 \sin^2(\psi_i + \alpha_i) + \sigma \psi_i^2 + \sigma \alpha_i^2 \cos^2(\psi_i + \alpha_i)}$$

$$\sigma y_{ri} = \sqrt{\sigma y_{Ai}^2 + \sigma d_i^2 \cos^2(\psi_i + \alpha_i) + \sigma \psi_i^2 + \sigma \alpha_i^2 \sin^2(\psi_i + \alpha_i)} \qquad (5)$$

$$\sigma x_{ri} y_{ri} = \sigma x_{Ari} y_{Ari} + \sigma d_i^2 \cos\psi_i \cos\psi_i - \sigma \psi_i^2 d_i^2 \cos^2 \psi_i$$

Determined uncertainties are directional errors of points coordinates. Graphical presentation of these uncertainties (Fig. 6) enables general errors evaluation and quick indication of sectors with significant errors magnitude. The standard uncertainties of distance (σd_i) and direction ($\sigma \lambda_i$) input values, describing vectors $\vec{P_A P_{ri}} = [d_i, \lambda_i]$ are sum of uncertainties coming from inaccuracy of the assessment of GNSS antenna location in the ship's coordinate frame (σd_{hfi}, $\sigma \alpha_{hfi}$) and also uncertainties propagated from ship's contour model approximation process respectively (σd_{apri}, $\sigma \alpha_{apri}$) (eqt.: 6, 7):

$$\sigma d_i = \sigma d_{hfi} + \sigma d_{apr.i} \qquad (6)$$

$$\sigma \alpha_i = \sigma \alpha_{hfi} + \sigma \alpha_{apr.i} \qquad (7)$$

The direction (α_i) of $\vec{P_A P_{ri}}$ vector (fig.1) is calculated according to formula (8):

$$\alpha_i = \arctan \frac{y_i}{x_i} \qquad (8)$$

where: x_i, y_i – coordinates of consecutive points of real model of ship's contour taken from ship's plan.

According to general rule of errors propagation after partial derivatives of indirectly measured values had been calculated, as a result obtained combined standard uncertainty α_I of $\vec{P_A P_{ri}}$ vector:

$$\sigma \alpha_{hfi} = \sqrt{\left(\frac{1}{\frac{y_i + x_i^2}{y_i}} \right)^2 \sigma x_{hf}^2 + \left(\frac{-x_i}{y_i^2 + x_i^2} \right)^2 \sigma y_{hf}^2} \qquad (9)$$

The distance d_i of $\vec{P_A P_{Ari}}$ vector is expressed by square root of relevant coordinates (x_i,y_i) sum, raised to a power of two:

$$d_i = \sqrt{x_i^2 + y_i^2} \qquad (10)$$

As a result of a differential calculus of equitation 10 with respect to (x_i, y_i) combined distance standard uncertainty (d_i) of $\vec{P_A P_{Ari}}$ vector was obtained:

$$\sigma d_{hf} = \sqrt{\frac{x_i^{\,2}\sigma x^2 + y_i^{\,2}\sigma y^2}{x_i^{\,2} + y_i^{\,2}}} \qquad (11)$$

After combined standard uncertainties of each point the ship's contour is built from had been provided to equitation 12, the formulas to calculate point's coordinates of ship's position uncertainty area in INS at a given confidence level was obtained:

$$x_{ni} = x_{ri} + c\sqrt{\sigma x_{Ari}^2 + \sigma d_i^2 \sin^2(\psi_i + \alpha_i) + \sigma \psi_i^{\,2} + \sigma \alpha_i^{\,2} \cos^2(\psi_i + \alpha_i)} \qquad (12)$$

$$y_{ni} = y_{ri} + c\sqrt{\sigma y_{Ari}^2 + \sigma d_i^2 \cos^2(\psi_i + \alpha_i) + \sigma \psi_i^{2} + \sigma \alpha_i^{\,2} \sin^2(\psi_i + \alpha_i)} \qquad (13)$$

The error ellipse is the most precise measure of ship's position and can be used to asses the accuracy of points the ship's contour is built from. It comes from her specific characteristics which are as follows (Gucma 2006): it is the only figure with constant probability density on her circumference, it enables to conclude from which direction the errors have greater values, parameters of ellipse allows to calculate directional errors, it gives the most probable location of ship's shape points among other figures with the same area.

Determining the geometrical centre, direction of axis and both semiaxis are essential in ellipse building process. The point the model ship's contour is built from and determined uncertainties of its coordinates were used to characterize the semiaxis and geometrical centres of error ellipses. The bigger semiaxis – ***a*** corresponds to direction error along X axis of cartesian reference frame. The smaller semiaxis – ***b*** corresponds to direction error along Y axis. Figure 2 presents hypothetical ellipse formed by 16 points described by parametric quotation: $x_j = a\cos\phi_j$, $y_j = b\sin\phi_j$ (ϕ_j – angle between X-axis and radius of j-th point of ellipse, *a, b* – length of bigger and smaller semiaxis of ellipse).

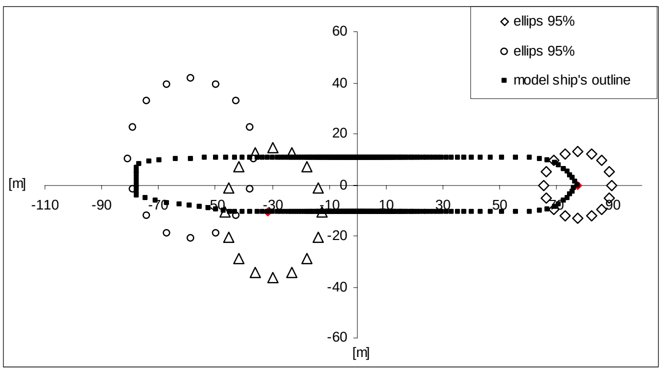

Fig. 2. The errors ellipses of chosen points the ship's shape outline is built from with semiaxes $a = \sigma x_{ri}$ i $b = \sigma y_{ri}$ formed in result of continuous line discretization into 16 points

Providing directional uncertainties to mentioned quotations obtained:

$$\begin{aligned} x_{ei} &= c\sigma x_{ri}\cos\phi_j + x_{ri} \\ y_{ei} &= c\sigma y_{ri}\sin\phi_j + y_{ri} \\ \phi_j &\in \langle 0; 2\pi) \end{aligned} \qquad (14)$$

where: x_{ei}, y_{ei} – consecutive points the ellipse is built from, x_{ri}, y_{ri} – calculated coordinates of consecutive points of ship's contour.

Having determined the ellipse errors for every points describing ship's contour the two-dimensional matrix of points P_i (x_i, y_i) is formed. The outline of the area covered by points of ellipses is found by searching through every sector with angle width Δ_α around the ship's shape. The extreme point in each sector is found on the basis of distances calculated between these points and reference point (geometrical centre of ship's shape). The extreme points create the limit of uncertainty area around the model ship at assumed confidence level (Fig. 3).

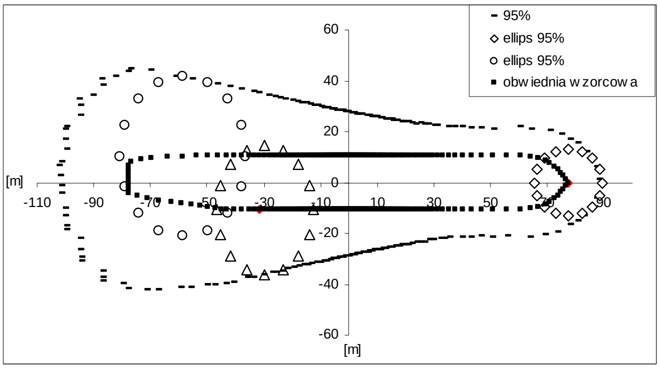

Fig. 3. The uncertainty area of ship's location around the model ship outline formed after extreme points of error ellipse had been found for 95% confidence level and antenna GNSS placed in fore part of the ship

Ship's location uncertainty area determining process is based on input data that do not change while calculations are being done. The dimensions of uncertainty area depend on heading the ship proceeds while maneuvering on research restricted area. That is why in practical approach the recorded ship's path coming from real experiment or simulated data are used. In order to achieve accurate results it is recommended to have this information inserted into model with the GNSS positioning frequency (1s). Directional errors of points the ship's contour is built from are determined for courses the ship is expected to proceed. In next step after statistical analysis the mean directional errors and errors at assigned confidence level are determined. This approach enables to take into consideration changeability the dimensions of uncertainty area depending on courses the ship is going to keep in real conditions. Picture 4 presents the ship Jan Śniadecki uncertainty area determined for input data assumed for pilot navigation system that uses EGNOS as a source of position and two synchronically working DGPS IALA receivers with reference station situated in Dziwnów. The reference DGPS antenna was placed in fore part of the ferry next to navigation bridge on starboard side.

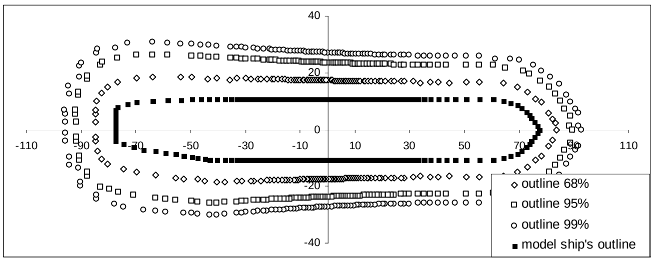

Fig. 4. The uncertainty area for Jan Śniadecki ferry, at given confidence levels

3 RESULTS OF EXPERIMENT

The analysis of research results was based on the evaluation of the size of uncertainty area occupied by the Jan Śniadecki ferry's contour, estimated with the use of uncertainty propagation theory (Tylor 1999). The magnitude of errors influencing ship's uncertainty area as the directly measured values was verified. The calculated error of waterline contour position where GNSS antenna was situated in geometric centre of ship's contour plane did not exceed 6m at the confidence level 0.95% (dashed line in fig. 5). In case when antenna was situated in the fore part of deck the error did not exceed 11m assuming directly measured errors as in tab. 1:

Table 1. Magnitude of directly measured errors

GNSS position error (DGPS IALA)	$\sigma x_{Ar} = \pm 0{,}96m, \sigma y_{Ar} = \pm 0{,}84m$
Accuracy of the assessment of antenna location	$\sigma x_{Hf} = \pm 1m, \sigma y_{Hf} = \pm 1m$
Heading error (2 sets of synchronized DGPS receivers)	$\sigma \psi = \pm 2{,}4^{\circ}$
Ship's model approximation error in i-th sector	$\sigma x_i = \pm 0{,}5m, \sigma y_i = \pm 0{,}4m$, $79 < i < 90, 90 < i < 101$ $\sigma x_i = \pm 0{,}3m, \sigma y_i = \pm 0{,}35m$, $255 < i < 270, 270 < i < 290$

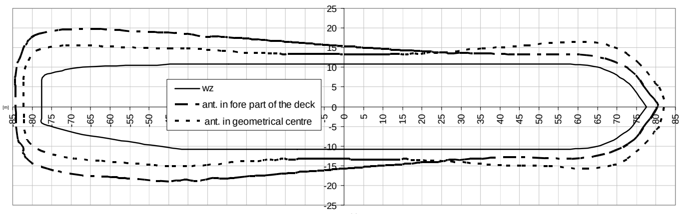

Fig. 5. Comparision of ship's location uncertainty area determined for INS using DGPS IALA position source and two synchronically working DGPS IALA receivers as a heading source with GNSS antenna situated in geometrical centre and out of it

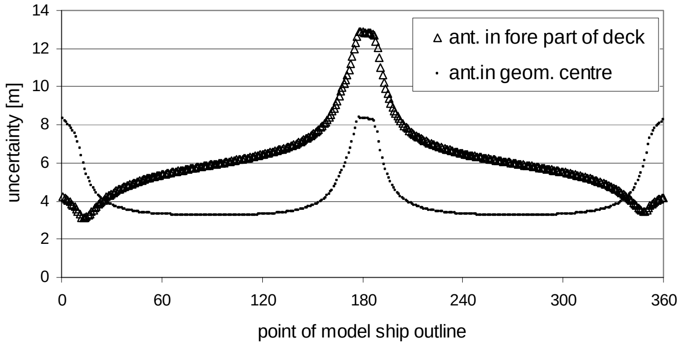

Fig. 6. Errors comparision of consecutive ship's outline points determined for INS using DGPS IALA position source and two synchronically working DGPS IALA receivers as a heading source with GNSS antenna situated in geometrical centre and out of it

The significant errors appear in bow and aft sectors of ship's contour due to GPS antenna is placed in the geometric centre of the contour. The calculated error of waterline contour position differs when GPS antenna is not situated in geometric centre of ship's contour plane (Tomczak 2006). Antenna reference – ship's (0,0) point was established 28m from the bow and 5m right from the centre line of the ship. The significant influence of heading error is clearly seen. The determined area is much wider in aft part of ship's shape and errors reach 11m at the confidence level 0.95 (dashed line in fig. 5) assuming directly measured errors as above.

4 CONCLUSIONS

The research has provided results that can be summarized as follows:

- Uncertainty error propagation theory may be applied to ship's location uncertainty area determination at assigned probability level in pilot navigation system,
- Worked out mathematical model of ship's location uncertainty area, allows to identify the position of ship's waterline with an error up to 6 metres at the confidence level 0.95 for the directly measured errors when GNSS antenna is placed in geometrical centre of ship's contour plane and in case when the GNSS antenna is shifted out of geometrical centre with error up to 11 metres at the confidence level 0.95,
- The determined uncertainty area strongly depends on GNSS antenna placing in relating to ship's coordinate frame when the directly measured errors remain unchanged,
- Worked out mathematical model of ship's location uncertainty area and the results obtained, can be used in the process of designing pilot navigation systems in respect of the ship visualization in a given area.

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